

Interferometry radii in heavy-ion collisions at $\sqrt{s} = 200\text{GeV}$ and 2.76TeV

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The expansion of the fireball created in Au-Au collisions at $\sqrt{s} = 200\text{GeV}$ and Pb-Pb collisions at 2.76TeV is modelled using the relativistic viscous hydrodynamics. The experimentally observed interferometry radii are well reproduced. Additional pre-equilibrium flow improves slightly the results for the lower energies studied.

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I. INTRODUCTION

The matter created in ultrarelativistic heavy-ion collisions undergoes a rapid expansion. During the expansion substantial transverse flow is generated modifying the transverse momentum spectra of emitted particles. The measured spectra and elliptic flow at the BNL Relativistic Heavy Ion Collider [1] can be described within relativistic hydrodynamic models [2, 3]. The hydrodynamic behavior of the bulk of the matter created in Pb-Pb collisions at $\sqrt{s} = 2.76\text{TeV}$ is confirmed in recent experiments at the CERN Large Hadron Collider (LHC) [4, 5]. The magnitude of the observed elliptic flow indicates that the dense matter behaves as an almost perfect, thermalized fluid, with small shear viscosity [6].

The size and the life-time of the source can be estimated from the Hanbury Brown-Twiss (HBT) radii extracted from Bose-Einstein correlations of identical particles [7]. The description of the HBT radii in dynamical models has been the subject of intensive studies. Many calculations lead to an overestimate of the ratio of R_{out} to R_{side} radii, which was termed as the HBT puzzle. Capturing correctly the values of the R_{side} and R_{long} radii in a model calculation amounts to a satisfactory description of the size and the life-time of the system. The radius R_{out} (and the ratio R_{out}/R_{side}) are more sensitive to the collective flow of the matter at the freeze-out. The same is true for the dependence of the radii on the momentum of the pion pair. Experimental observations yield $R_{out}/R_{side} \simeq 1$, contrary to expectations for a system undergoing a first order phase transition [8]. Reversing this argument, the experimental data indicate that the equation of state of the hot matter is hard, without a soft point [9, 10]. Other effects that influence the ratio R_{out}/R_{side} are the shear viscosity [10, 11], the partial chemical equilibrium [12], the early start of the hydrodynamic expansion (0.1-0.25fm/c) [9, 10] or the presence of some pre-equilibrium flow [13].

We simulate central heavy-ion collisions at the top RHIC and LHC energies using relativistic viscous hydro-

dynamics, with or without pre-equilibrium flow. We calculate the transverse momentum spectra and HBT radii. We obtain a satisfactory agreement with the experiments, the additional initial flow improves the R_{out}/R_{side} ratio at RHIC energies but has a smaller effect for Pb-Pb collisions at the LHC.

II. HYDRODYNAMIC MODEL

We use the second order relativistic viscous hydrodynamics [14] to model the expansion of fireball. In relativistic viscous hydrodynamics the energy-momentum tensor is

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} + \pi^{\mu\nu} + \Pi\Delta^{\mu\nu}, \quad (1)$$

with stress corrections from shear $\pi^{\mu\nu}$ and bulk $\Pi\Delta^{\mu\nu}$ viscosities, $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$. The stress corrections are the solutions of the second order relativistic viscous hydrodynamic equations [6, 11, 14, 15]

$$\Delta^{\mu\alpha}\Delta^{\nu\beta}u^\gamma\partial_\gamma\pi_{\alpha\beta} = \frac{2\eta\sigma^{\mu\nu} - \pi^{\mu\nu}}{\tau_\pi} - \frac{1}{2}\pi^{\mu\nu}\frac{\eta T}{\tau_\pi}\partial_\alpha\left(\frac{\tau_\pi u^\alpha}{\eta T}\right) \quad (2)$$

and

$$u^\gamma\partial_\gamma\Pi = \frac{-\zeta\partial_\gamma u^\gamma - \Pi}{\tau_\Pi} - \frac{1}{2}\Pi\frac{\zeta T}{\tau_\Pi}\partial_\alpha\left(\frac{\tau_\Pi u^\alpha}{\zeta T}\right), \quad (3)$$

with

$$\sigma_{\alpha\beta} = \frac{1}{2}\left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{3}\Delta_{\alpha\beta}\partial_\mu u^\mu\right), \quad (4)$$

η and ζ are the shear and bulk viscosity coefficients and two relaxation times τ_π and τ_Π appear. We use the Navier-Stokes initial conditions for the stress tensor $2\pi^{xx}(\tau_0) = 2\pi^{yy}(\tau_0) = -\pi^{zz}(\tau_0) = 4\eta/3\tau_0$ corresponding to the longitudinal Bjorken flow and $\Pi(\tau_0) = 0$.

The initial profile of the entropy density at the impact parameter b is obtained from the Glauber model

$$s(x, y, b) = s_0 \frac{(1 - \alpha)\rho_{WN}(x, y, b) + 2\alpha\rho_{bin}(x, y, b)}{(1 - \alpha)\rho_{WN}(0, 0, 0) + 2\alpha\rho_{bin}(0, 0, 0)}, \quad (5)$$

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where ρ_{WN} and ρ_B are the densities of wounded nucleons and binary collisions respectively. The optical Glauber model densities are obtained with Wood-Saxon densities for the colliding nuclei $\rho_{WS}(r) = \rho_0 / (\exp((r - R_a)/a) + 1)$ ($\rho_0 = 0.169\text{fm}^{-3}$, $R_a = 6.38\text{fm}$, $a = 0.535\text{fm}$, $b = 2.1\text{fm}$ for Au and $\rho_0 = 0.169\text{fm}^{-3}$, $R_a = 6.48\text{fm}$, $a = 0.535\text{fm}$, $b = 2.2\text{fm}$ for Pb) and the nucleon-nucleon cross sections are 42mb and 62mb at $\sqrt{s} = 200\text{GeV}$ and 2.76TeV respectively. The contribution of binary collisions is $\alpha = 0.145$ at RHIC energies [16]. The centrality dependence of the charged particle multiplicity in Pb-Pb collisions at the LHC [17] is used to fix $\alpha = 0.15$ at the higher energy. Such a value of the parameter α leads to an apparent overprediction of the multiplicity in central Pb-Pb collisions [18, 19], but this is due to the incorrect value of the reference multiplicity in proton-proton collisions used in Ref. [18]. We use a realistic equation of state interpolating between the lattice QCD data at high temperatures and an equation of state of a gas of hadrons at lower temperatures [20]. The use of an equation of state without a soft point is essential in reproducing the femtoscopy data [9, 10].

The initial time for the hydrodynamic evolution is $\tau_0 = 0.6\text{fm}/c$. The corrections to the pressure from shear viscosity $2\eta/3\tau_0$ to the transverse pressure at the initial time are of the order of 20%, hence the application of viscous hydrodynamics is justified. On the other hand, the starting time of the hydrodynamic evolution should be defined by the early thermalization mechanisms that are not well understood. Moreover, some early flow can be generated before the start of the hydrodynamic evolution. The pre-equilibrium transverse flow in hydrodynamic models has been discussed by several authors [13, 21, 22]. The mechanism generating the initial flow is not known, and the amount of the pre-equilibrium flow is a parameter of the calculation. Ref. [22] gives a general result for the transverse flow generated in the initial stage of the reaction, which could serve as an upper bound for the pre-equilibrium flow generated in the expansion at the early stage with boost invariance, starting at zero time.

To estimate the effects of the collective evolution of the system before τ_0 and of the choice of the value of τ_0 , we compare two different calculation. The first scenario assumes that the evolution starts at $\tau_0 = 0.6\text{fm}/c$ with zero initial flow and in the second scenario the initial flow is taken in the form of the universal flow for the early transverse acceleration [22]. For a traceless energy-momentum tensor the components of the energy momentum tensor take a universal form

$$\frac{T^{0i}}{T^{00}} = -\frac{\partial_i T^{00}}{2T^{00}}\tau_0 \quad (6)$$

at the time τ_0 .

Starting from τ_0 the evolution is governed by the relativistic viscous hydrodynamics with a realistic equation of state. The initial energy density and pressure are given by the Glauber profile of the entropy density with corrections from shear viscosity given by the Navier-Stokes

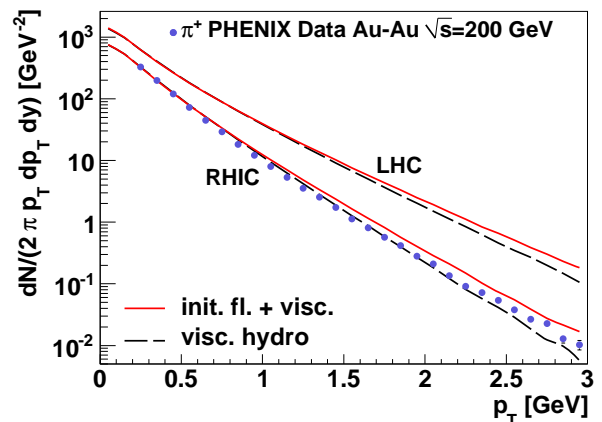


FIG. 1. (Color online) Transverse momentum spectra of π^+ in central collisions. The upper lines are for Pb-Pb collisions at $\sqrt{s} = 2.76\text{TeV}$ and the lower lines for Au-Au collisions at 200GeV. The dashed lines are for standard initial conditions and the solid lines are for the evolution starting with the pre-equilibrium flow, all calculations in the relativistic viscous hydrodynamics; data for Au-Au collisions are from the PHENIX Collaboration [23].

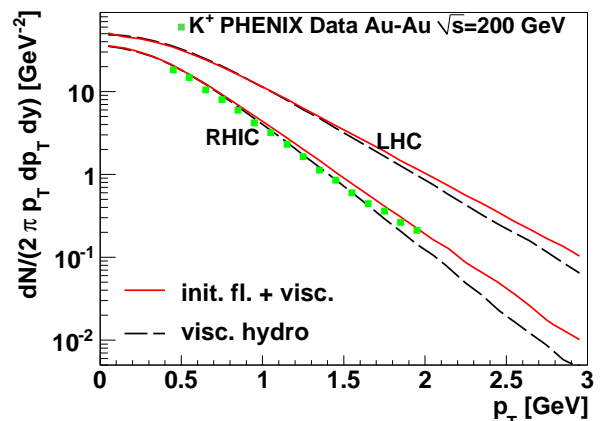


FIG. 2. (Color online) Same as Fig. 1 but for K^+ .

value used as initial conditions for the second order Israel-Stewart equations. The velocity profile at τ_0 is matched to reproduce the ratio T^{0i}/T^{00} predicted from the universality argument (6). The effects of switching to a realistic equation of state and of the shear viscosity corrections after τ_0 on the velocity field are small in the core of the fireball as compared to using a perfect fluid with the ultrarelativistic gas equation of state.

III. RESULTS

We set the ratio of the viscosity coefficients to the entropy to $\eta/s = 1/4\pi$ for the shear viscosity and $\zeta/s = 0.04$ for the bulk viscosity, $\tau_\pi = \tau_\Pi = \frac{3\eta}{T_s}$. The bulk viscosity is present only in the hadronic phase [11].

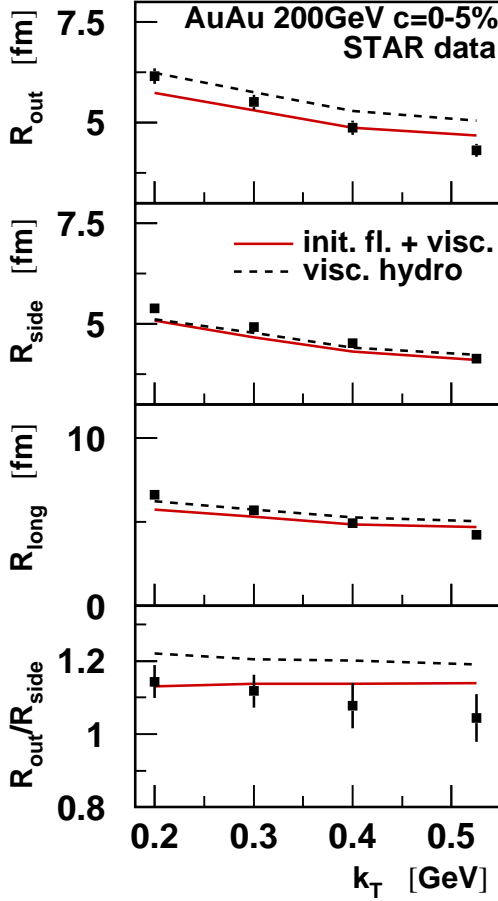


FIG. 3. (Color online) STAR Collaboration data [25] (squares) for the HBT radii in central Au-Au collisions at $\sqrt{s} = 200\text{GeV}$ compared to the results from viscous hydrodynamics with standard (dashed lines) and pre-equilibrium flow initial conditions (solid lines).

The statistical emission and resonance decays are performed using the THERMINATOR program [24], at the freeze-out temperatures of 140MeV and 150MeV for the calculations without and with pre-equilibrium flow respectively. The freeze-out with finite shear and bulk viscosities in the hadronic phase captures the main characteristics of the dissipative hadronic rescattering phase at the end of the expansion [11]. The entropy density at the center of the fireball s_0 corresponds to a temperature of 400 and 520MeV at RHIC and LHC energies respectively.

In Figs. 1 and 2 are shown the transverse momentum spectra of pions and kaons. To compensate for the additional transverse flow, the freeze-out temperature is increased to 150MeV for calculations including the initial flow. The introduction of the initial flow leads to some hardening of the spectra as compared to the standard initial conditions. However, both calculations are close to the experimental data. The transverse momen-

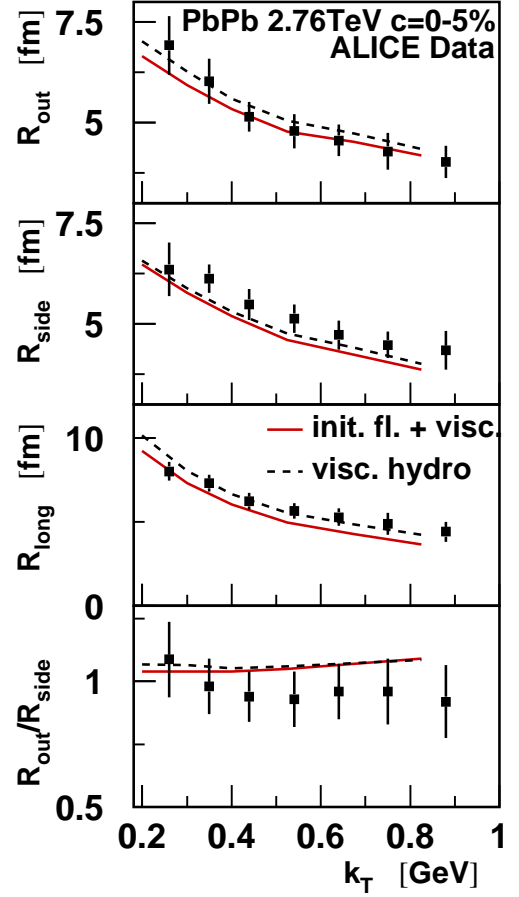


FIG. 4. (Color online) ALICE Collaboration data [5] (squares) for the HBT radii in central Pb-Pb collisions at $\sqrt{s} = 2.76\text{TeV}$ compared to the results from viscous hydrodynamics with standard (dashed lines) and pre-equilibrium flow initial conditions (solid lines).

tum spectra at LHC energies are harder and the relative importance of the pre-equilibrium flow is smaller than at RHIC. The system lives longer and most of the transverse flow is generated in the hydrodynamic expansion of an almost perfect fluid.

For events generated in THERMINATOR the correlation function is constructed from same-event and mixed-event pion pairs [26]. The three dimensional momentum correlation functions are fitted using a Gaussian parameterization and the three HBT radii are extracted. The dependences of the three radii and of the ratio R_{out}/R_{side} on the momentum of the pion pair are shown in Fig. 3 together with the experimental results of the STAR Collaboration [25]. The HBT radii are reproduced in the viscous hydrodynamic calculation with standard initial conditions to within 8-15%, depending on the pair momentum. The additional pre-equilibrium flow (solid lines) brings the results closer to the data, reducing the discrepancy. The satisfactory description of the ratio

R_{out}/R_{side} shows that the collective flow generated in the model is realistic. The values of the three radii are close to the data, which indicates that the size and the life-time of the source are well described in the model.

The source created in Pb-Pb collisions at the LHC is larger and shows a stronger collective flow (Fig. 4). The viscous hydrodynamic calculation reproduces the measured values of the radii to within 14%. The inclusion of the initial flow at LHC energies does not change the agreement with the data. The earlier freeze-out for the calculation with initial flow leads to a reduction of the values of the radii, but the ratio R_{out}/R_{side} is not modified significantly. We notice that the expansion at the LHC is dominated by the hydrodynamic stage, with a relatively smaller contribution from the pre-equilibrium phase. The strong flow observed in the femtoscopy data is quantitatively well described by the model calculation. The size of the fireball at the freeze-out increases with the increasing multiplicity, as seen in the data. In the simulation the freeze-out happens at $\tau \simeq 12\text{-}13\text{fm}/c$.

IV. CONCLUSIONS

We make hydrodynamic calculations for central heavy-ion collisions at $\sqrt{s} = 200\text{GeV}$ (RHIC) and 2.76TeV (LHC). The fireball expansion is modelled using relativis-

tic viscous hydrodynamics with the shear viscosity coefficient $\eta/s = 1/4\pi$, and, in the hadronic phase, the bulk viscosity of $\zeta/s = 0.04$ is added. Calculations starting without initial flow at $\tau_0 = 0.6\text{fm}/c$ describe the transverse pion and kaon spectra at RHIC energies. The HBT radii at both colliders are reproduced to within 15%. The presence of an additional transverse flow in the initial state of the hydrodynamic expansion improves the description of the HBT data at RHIC energies. The deviations of the simulations from the data are of the order of the systematic errors quoted by the experimental Collaborations.

The paper presents the first quantitative description of the recently released ALICE Collaboration data on the HBT interferometry in Pb-Pb collisions at the highest available energy [5]. The relativistic viscous hydrodynamics gives a satisfactory agreement with the measurements. The effect of the pre-equilibrium flow on the R_{out}/R_{side} ratio is small.

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